Method of 3D Mesh Reconstruction from Point Cloud
Using Elementary Vector and Geometry Analysis

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Abstract—The method of three dimensional mesh reconstruction is used to supplementing the 3D reconstruction or 3D scanning method, which only provide with point cloud in 3D model. However, the 3D models in mesh mode illustrate clearer appearance and can be used to obtain more information. In addition, the 3D reconstruction has been previously developed by employing the digital fringe pattern projection where the output was formed in the point cloud. Therefore, this paper presents the computerized algorithms to construct the 3D mesh from the point cloud. The algorithm can be done by firstly dividing the point cloud into small volume for parallel calculation. Next, the points in each small volume are connected, which becomes the submesh and then integrated them together. The submesh in each small volume is produced by creating the initial triangle and then expanding triangular construction continually from each side (base) which has been previously generated. The expanded triangles must be satisfied some certain constrains. The point cloud which is filtered from the STL file format was tested using proposed method. The mesh reconstruction is shown with reasonable result.

Keywords—point cloud; mesh; 3D surface; surface reconstruction; reconstruction algorithms;

I. INTRODUCTION

In reconstruction of the three dimensional (3D) model, almost final output from imaging equipments such as the stereo camera, the camera with digital fringe pattern projection, the whole body scanner and so on, is in 3D point cloud format. The 3D point cloud shows only the approximating shape. Information such as the area of the surface and clearer appearance are not provided. Therefore, for more efficient 3D model analysis, the reconstruction of 3D mesh process is required. The mesh is composed of connecting triangles. Each triangle is a unit to construct any polygon.

There have been tremendous amount of the studies in the 3D model reconstruction. For examples, in 2007, Stefano Ferrari and et.al approached to reduce and filter the 3D point cloud by using enhanced vector quantization (EVQ) technique[1]. This process must be completed before the mesh reconstruction process is operated. In 2009, Andrei C. Jalba and Jos B.T.M. Roerdink proposed the physically based technique for surface reconstruction which employs regularized membrane potentials evaluated on a volumetric grid [2]. This method approached to produce smooth surface output although the input data were noisy. This method had limitation that it was not geometrically adaptive. In 2010, Fengxia Li and et.al reduced the point cloud by uniform sampling in terms of the fitting line via nonlinear least square, and then applied Radial Basis Function Neural Network (RBFNN) technique for filling the holes, which were the defect on the surface obtained by the scanner [3]. However, the output was still point cloud. In 2011, Kun Zhou and et.al proposed the octree construction on the GPU which performed the existent Poisson surface reconstruction method [4]-[5].

This paper presents a new method of 3D mesh reconstruction from the point cloud data. The proposed method is based on elementary vector analysis and geometry. This algorithm can reduce processing time because the parallel calculation could be performed. The program is written by using C++ Builder and OpenGL library for 3D display. The input data is acquired by two sources. Firstly, the instant point cloud is filtered from the STL file format (Only vertex data, mesh and normal vector are not included). Secondly, the point cloud obtained from 3D reconstruction using digital fringe pattern projection technique [6] which has been studied previously. For further study, the 3D reconstruction using digital fringe pattern projection and mesh reconstruction from this article will be combined in order to complete the 3D reconstruction system.

II. METHOD

A rectangular solid is constructed to cover the whole point cloud before it is divided into smaller cubes. Each cube contains some parts of the point cloud as shown in Fig. 1. In each cube, the points are connected so that they become
component of mesh, a mesh formed from triangles. The algorithm creates the sequences of triangle construction. Each sequence starts with initiating the first triangle and then creating a chain of triangle construction which expand under the direction information of the previous construction. The algorithm can be explained in more details as follows.

A. The Construction of the First Triangle in each Chain Construction inside any Cube

In each cube, any point which has never been a component of other submeshes built from previous chain construction (the degree of such point is defined to be zero), can be selected to be an initial point (point A). We then calculate the distance between point A and other points that have degree zero. Next, the Quick Sort technique [7] is applied to find the shortest distance between point A and those points. Let B be the point that gives the minimum distance to point A. Then the first side of the triangle is obtained, AB. For completing the initial triangle, the third vertex C must be chosen so that it satisfies all of the following constrains:

- First, \( C \) satisfies the following inequality:

\[
\max(|AB|, |AC|, |BC|) < |AB| + |AC| + |BC| - \max(|AB|, |AC|, |BC|)
\]  

where \( |AB|, |AC| \) and \( |BC| \) are the lengths of the edges \( AB, AC \) and \( BC \) respectively. Moreover, the point \( C \) must give the minimum value \( d_C \) where

\[
d_C = |AB| + |AC| + |BC| - \min(|AB|, |AC|, |BC|)
\]

Figure 2 illustrates how the point \( C \) is chosen so that the value \( d_C \) is minimum. The Quick Sort Technique is applied to find the point that gives the minimum value \( d_C \).

- The initial triangle containing the point \( C \) cannot overlap to any existed triangle. The notion of overlapping is explained by the following consideration. There are two types of overlapping. The first type occurs when at least one side containing the point \( C \) locates between the adjacent sides of the triangle that have been previously created. For convenient citation, Type I Overlap will be called in this situation. The following procedure, which uses only the concept of the dot product and the cross product, will be used to examine the Type I Overlap. This method is shown in Fig. 3.

The second type of overlapping happens if the angle of the initial triangle at the point \( C \) contains the angle of another already existed triangle where \( C \) is also the vertex of this triangle. For succinct citation, Type II Overlap is called in this incident. Again we use the knowledge of the geometry of the dot product and the cross product for checking the Type II Overlap. This method is demonstrated in Fig. 4.
number of times that each side used to construct a triangle, duo vectors locate the angle on each vertex, the degree of each vertex and the direction for constructing the next triangle on each side of the initial triangle must be evaluated. The direction on each side is named by the Expanding vector, \( \hat{e} \). The vector \( \hat{e} \) is the unit vector calculated by the following equation:

\[
\hat{e} = \frac{\vec{u} \times \vec{w}}{\|\vec{u} \times \vec{w}\|}
\]  

(3)

where \( \vec{v} \) is the unit vector in the direction of the side and \( \vec{w} \) is the vector of another side sharing the same vertex as shown in Fig. 5

![Figure 5](image)

**Figure 5.** (a) Creating Expanding Vector on the edge \( \overline{AB} \), parameters \( \vec{v} \) and \( \vec{w} \) are required where \( \vec{v} \) is the unit vector of the vector \( \overline{AB} \) and \( \vec{w} \) is the vector \( \overline{AC} \). By the expression (3), the Expanding Vector \( \hat{e}_{AB} \) is obtained. (b) The Expanding Vectors on each side of the initial triangle are generated.

**B. The Construction of the Sequence of Triangles in each Chain Construction inside each Cube**

Each side of the initial triangle which is not a common side is the source of the subsequent triangle construction. It is going to search the third point for completing a nearby triangle (if the triangle containing the mentioned side exists) on the following conditions:

- The vector from the considered edge to the third point and the Expanding Vector of the edge are aligned on the same side of the edge. This situation is illustrated in Fig. 6.

The notion that the vector \( \vec{u} \) from the considered edge to the inspected point and the Expanding Vector \( \hat{e} \) of the edge aligns on the same side of the edge if the angle between them is acute. This condition can be checked by the following simple inequality:

\[
\vec{u} \cdot \hat{e} > 0
\]

(4)

- The triangle containing the third point as the vertex must have the angles at the base between 5 to 120 degree. Moreover, the two new produced edges are not allowed if at least one of these two edges is the common side of other existed triangles as shown in fig.

![Figure 7](image)

**Figure 7:** The point \( C \) in both (a) and (b) agree with the first condition of the third point selection. Moreover, in (a) and (b) the vector \( \vec{u} \) from \( \overline{AB} \) to point \( C \) and the Expanding vector \( \hat{e}_{AB} \) align on the same side of \( \overline{AB} \) and the triangle \( ABC \) has the angles \( \theta \) and \( \psi \) on the base \( \overline{AB} \) between 5 to 120 degree. However, the contrast between point \( C \) in (a) and (b) is that the side \( \overline{AC} \) in (a) is the common side to another existed triangle while the side \( \overline{AC} \) in (b) is the common side to other two existed triangles. In these situations (a) and (b), only (a) will be legitimate.

- The third point must satisfy all the conditions as we required for the point \( C \) in the construction of the initial triangle (see part A). In addition, the Type I Overlap and the Type II Overlap must be checked at every corner of the triangle not only at the third point.

After the third point is obtained, we get the new triangle and its information (the number of each side used to construction a triangle, duo vectors locate the angle on each vertex, the degree of each vertex, and the Expanding vector of each side) are recorded. There are recently two constructed edges of this new triangle that we will use to construct the next triangles under the same constrains. Now, repeating the same procedure until we don’t have any additional triangle generated. Then the construction of the chain of the triangles now is finished.

**C. The Submesh Integration**

The parallel processing technology is applied to the chains of triangles construction in each cube. After the process is completed, the collection of the chains in each cube becomes a submesh contained in that cube. The last process is compiling all submeshes together. The process begins with the cube located on the far left corner (our sight) of the rectangular solid as shown in Fig. 8 (a). Then the single edges of the submeshes are selected to be a source of the triangle construction. Each single edge in this cube will search for its third points from the adjacent cubes to form additional triangles (see Fig. 8 (a)).
The rules of the triangle construction for connecting the submeshes together are the same rules as the construction of the triangles after the initial triangle obtained (see B). After each submesh is connected to its seven adjacent submeshes (may be less than seven if the considered cube position on the rim), the mesh is complete.

III. EXPERIMENTAL RESULT AND DISCUSSION

The result of the 3D mesh reconstruction algorithm can be seen in Fig. 9.

The point cloud of real world object which is named Laurana was tested using our proposed method. The result shows the satisfying mesh reconstruction.

The second constrain in section II (B), the angles on the base were fixed at 5 to 120 in these experiments. If the interval is decreased such as 5 to 80, the time taken in processing would be increased and there were holes occurred on the mesh, but the mistake in the triangle construction would be reduced.

Hence, an appropriate angle should be chosen from the experiments.

The proposed algorithm still needs to be improved in terms of the memory management for displaying on the screen.

For further study, the 3D reconstruction using digital fringe pattern projection and mesh reconstruction from this paper will be combined in order to complete the 3D reconstruction system. In addition, the calculation of the normal vector of triangular face will be developing to obtain the 3D surface.

IV. CONCLUSION

The algorithm of 3D mesh reconstruction from point cloud data has been presented. The algorithm can be done by dividing the point cloud into small volume for parallel calculation, connecting the point in each small volume become the submesh and then integrating them together. The submesh in each small volume was produced by creating the initiate triangle then expanding triangular construction continually from each side (base) which had been generated. The later triangle must be under the conditions: located to be related with the Expand Vector, retrieved the angles on base 5 to 120 degree, obtained minimum value \( d_c \), taken sides had been constructed the triangles not exceed 2 times and did not come across Type I and Type II Overlap.

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