DENTAL TOOL CALIBRATION USING POINT-CLOUD-TO-POINT-CLOUD TECHNIQUE WITH THE LEAST-SQUARE SOLUTION: TOWARD DEVELOPMENT OF A DENTAL NAVIGATION SYSTEM

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This paper presents a dental tool calibration based on point-cloud-to-point-cloud rigid transformations and singular value decomposition (SVD). The system is a part of CIS interventions in dental implant navigation system. The system is composed of optical tracking, image processing and parameter operation parts setting up with the least square problem to find the transformation components. Results show higher accuracy compare to the standard tool-tip calibration algorithms.

1. INTRODUCTION
Dental implant becomes a common method of tooth root replacement used in prosthetic dentistry. Computerize navigation system on a pre-surgical plan is offered to minimize potential risk of damage to critical anatomic structures of patients. The methodology based on CIS interventions includes preoperative and intraoperative procedures. The preoperative surgery required the 3D views of raw images obtained from CT scanning, while the target pathway guidance is beforehand generated. Intraoperative surgery includes tracking systems, a stereo scope calibration, and a set of special designed reference markers attached on the dental hand-piece tool and patient’s jaw. They are associatively registered together in the system. Dental tool tip calibrating is basically an important procedure to determine the relation between the hand-piece tool tip and hand-piece’s markers which can be formed as vector $b_{tip}$. With the transferring coordinates from preoperative data to reality, this parameter is a part of components in typical registration problem. A high accuracy is required, and this relation is arranged by Point-cloud-to-point-cloud rigid transformations and singular value decomposition (SVD) for minimizing rigid registration errors [1, 2]. The proposed procedure is to use the pointing device or hand-piece to touch the point on the pivot and the transformation matrix is calculated every time when it moves to the new position while the tool tip stays at the same point. The transformation matrix is formed by 3x3 rotation matrix, and translation vector interpreting of actually translating the point in space.

Figure 1. Pointing Device Calibration Technique
2. METHOD AND ALGORITHM

For each measurement \( k \), we have \( \mathbf{b}_{\text{post}} = \mathbf{R}_b \mathbf{b}_{\text{tip}} + \mathbf{p}_r \). With new arrangement, \( \mathbf{R}_b \mathbf{b}_{\text{tip}} - \mathbf{b}_{\text{post}} = -\mathbf{p}_r \).

Set up a least square problem:

\[
\begin{bmatrix}
R - I
\end{bmatrix}
\begin{bmatrix}
\mathbf{b}_{\text{tip}} - \mathbf{b}_{\text{post}}
\end{bmatrix}
= 
\begin{bmatrix}
M
\end{bmatrix}
\]

This can be represented as \( \tilde{A} \mathbf{b} = \tilde{p} \) and the vector can be written as \( \mathbf{b} = (\tilde{A}^T \tilde{A})^{-1} \tilde{A}^T \tilde{p} \), where \( (\tilde{A}^T \tilde{A})^{-1} \) is pseudo inversion. Point-cloud-to-point-cloud rigid transformations is typically technique to calculate the transformation matrix by using a set of points \( \{a_i\} \) in one coordinate system and another set of points \( \{b_i\} \) in a second coordinate system to find \( [\mathbf{R}, \mathbf{p}] \) that minimize the least square of rigid registration errors,

\[
\Sigma' = \sum_{i} \| \mathbf{R} \cdot \mathbf{a}_i + \mathbf{p} - \mathbf{b}_i \|,
\]

where \( \mathbf{R} \) is a 3x3 rotation matrix, \( \mathbf{p} \) is a translation vector [1,2].

2.1 System Overview

Figure 2 illustrates the system configuration. The calibrated stereo camera with infrared filter is used to retrieve image shown in Figure 3. The Image Acquisition has been implemented to capture monochrome image through MATLAB\(^\text{®} \). Moreover, the stereo-snapshot of left and right images for each measurement, \( k \) have been stored, each image is processed to obtain the 3D coordinate of marker positions. Thresholding and morphological image processing techniques have been performed to complete the structuring elements of object region. All centroids of marker region in left and right stereo images, and intrinsic and extrinsic stereo parameters are input into stereo triangulation function to find the 3D positions which are equivalent to the world coordinate system. With these two sets of point cloud, the transformation matrix is calculated by SVD.

2.2 Optical Tracking

2.2.1 Stereo Camera Calibration: The stereo camera calibration has been done by Camera Calibration Toolbox for MATLAB\(^\text{®} \) based on planar checkerboard pairs of corresponding left and right images. After calibration, the list of intrinsic and extrinsic parameters are stored and used for computing the 3D location of a set of points given their left and right image projections. This process is known as stereo triangulation [3].

2.2.2 Tracking Device: The active optical tracking using IR-LEDs is used as markers or tracked objects in the system. The two set markers are utilized as reference frame, \( \mathbf{F}_{\text{ref}} \), and the other is on the top of the tool holder which called the pointing frame, \( \mathbf{F}_{\text{ptr}} \). With these two sets of markers, they represent the two set of point clouds \( \{a_i\} \) and \( \{b_i\} \).
2.2.3 Image Acquisition: The image acquisition stage is the first stage of the vision system. The image is satisfactorily obtained by software which implemented on Digiclops® and Triclops, Bumblebee Library to capture monochrome images through MATLAB®. The left and right images are necessary to be captured simultaneously in each pool of measurement.

2.3 Image Processing

If the images have been acquired satisfactorily, then the intended tasks are achievable with the aid of some form of image enhancement. The acquired images are changed to binary image by thresholding technique in order to separate an object in the image from the background, and use the combination of dilation and erosion, together with opening and closing to enhance the structure of the interesting region. With these improved regions in images, the centroids of the objects present in both left and right images are calculated. Then the positions of objects express the geometry in terms of 3D coordinates are computed by stereo triangulation technique [4].

2.4 Parameter Operation

Table 1 illustrates algorithm to calculate the transformation matrix using point-cloud-to-point-cloud technique. To find rotation matrix $R$ that minimize $\Sigma^2$, the noniterative algorithm involves the singular value decomposition (SVD) of a 3x3 matrix is utilized.

3. EXPERIMENT

3.1 Tools and Equipments

1. Bumblebee2® stereo vision camera system (BB2-BW-60), monochrome, 640x480 at 48 FPS, 6.0 mm focal length lens, two 1/3" progressive scan CCD sensors, and both the left and right images to a PC transmission via an IEEE-1394 interface.
2. Infrared filter covers the stereo cameras
3. Two markers with 4-IR-LEDs for each
4. Pivot
5. Tool Holder
6. MATLAB® program for calculation
7. Camera Calibration Toolbox for MATLAB®

3.2 Simulation and Result

Computer simulations have been carried out on MATLAB®. The result has been compared with respect to the normal pointing device calibration. 17 measurements of pointing changes with
position are performed in the experiment. The left and right images have been loaded by optical tracking component and have been processed by image processing component. The positions of objects with respect to three-dimensional coordinates are performed as set of point clouds \{a_i\} and \{b_i\}. \([R, p]\) is calculated by the least square problem setting up in order to compute \(b_{tip} = \hat{A}^{-1} \hat{A}^T \hat{A} \hat{p}\). This result has been compared with the standard pointing device calibration and found that the expected result differs from the standard method with the error 0.7700 %.

### 4. CONCLUSION AND FUTURE PROSPECT

This paper gives a systemic introduction about our ongoing project, the dental navigation system. In this work, we construct and fulfill some simple experiments on tool tip calibration with Point-Cloud-to-Point-Cloud Technique and the Least-Square Solution which obtain 0.7700 % error. The experiment acquired the information of tracking device, image acquisition and image processing algorithm. Future tasks consist of the integrated system with preoperative surgery and intraoperative tracking systems. A series of experiments of overall system will be carried out in order to validate the performance of the system and algorithm.

### 5. ACKNOWLEDGEMENT

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### 6. REFERENCES


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<th>Table 1. Algorithm to find rigid transformation and minimized rotation</th>
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<td><strong>Algorithm to find rigid transformations</strong></td>
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<tr>
<td><strong>Step 1:</strong> Compute centroid of each point cloud</td>
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<tr>
<td>( \bar{a} = \frac{1}{N} \sum_i a_i )</td>
</tr>
<tr>
<td>( \hat{a}_i = a_i - \bar{a} )</td>
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<tr>
<td><strong>Step 2:</strong> Find ( R ) that minimizes</td>
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<tr>
<td>( \sum_i (R \hat{a}_i - \hat{b}_i)^2 )</td>
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<tr>
<td><strong>Step 3:</strong> Find ( \hat{p} )</td>
</tr>
<tr>
<td>( \hat{p} = \bar{b} - R \bar{a} )</td>
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<tr>
<td><strong>Step 4:</strong> Desired transformation is</td>
</tr>
<tr>
<td>( F = \text{Frame}(R, \hat{p}) )</td>
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